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THERMAL PROCESSES IN CAPILLARY-POROUS BODIES WITH INTERNAL AND EXTERNAL HEAT SOURCES

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The problem of heating by internal energy sources a capillary-porous body in a medium with an elevated temperature is solved. The special features of the processes of heat and moisture transfer for different relations of the powers of the internal and external heating are analyzed. Simplified equations that describe the course of these processes are derived.

The thermal processes that occur in capillary-porous bodies have long been an object of scientific research. This is due both to their being of physical interest and to the great significance of these processes for the practical problem of drying wood, ceramics, agricultural products, and other materials.

A great contribution to the solution of this problem has been made by the works of A. V. Luikov and his school [1-6], in which consideration is given to many theoretical and practical questions connected with the transfer of energy and material in the process of drying. However, in them comparatively little attention is paid to the problem of heating bodies by electromagnetic radiation. In the meantime, much attention has been paid recently to the use of superhigh-frequency heating for the drying of various materials. This is due to the higher capacity of superhigh-frequency installations as compared to conventional thermal drying apparatuses and the better quality of the materials dried, since in the process of superhigh-frequency heating they are heated uniformly throughout the volume rather than in the surface layer, as during the use of conventional methods of drying.

In this work, we solved the problem of combined heating of a capillary-porous body by internal and external heat sources, which models the joint action of a superhigh-frequency field and an elevated temperature of the ambient medium.

Let us consider the heating of an infinite plane-parallel plate. This is a good approximation for many practical cases – drying of wood, ceramics, and certain foodstuffs (potatoes, apples). The results of the solution can also be employed for an approximate analysis of thermal processes in bodies of a different shape, taking into account the fact that in a cylinder their rate is higher by approximately a factor of 1.5 while in a sphere it is higher by a factor of 3 than in a plate with the same transverse dimensions [2].

Let the x axis be directed perpendicular to the plate surfaces and the origin of coordinates be in the middle between them, so that the surfaces are located at $x = -R$ and $x = R$. The initial temperature of the plate is T_0, the initial moisture content is U_0. At the instant $t = 0$ the temperature of the ambient medium increases abruptly to the value T_{amb} and then it remains constant. Starting with this instant of time heat sources whose volume density is equal to Q begin to act inside the plate.

For superhigh-frequency heating the density of the power of the internal heat sources is found from the formula [1]

$$Q = 0.555 \cdot 10^{-12} E^2 f \varepsilon' \tan \delta \ \ [\mathrm{W/cm^3}] ,$$

where E is the electric-field strength, V/cm; f is the frequency, Hz; ε' is the real part of the relative dielectric permittivity of the substance; $\tan \delta$ is the dielectric-loss tangent in the substance.

The heat and mass transfer equations and the boundary and initial conditions are written as

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$$\frac{\partial T}{\partial t} = a_q \frac{\partial^2 T}{\partial x^2} + \frac{\varepsilon r}{c} \frac{\partial U}{\partial t} + \frac{Q}{c\rho} , \tag{1}$$

$$\frac{\partial U}{\partial t} = a_m \frac{\partial^2 U}{\partial x^2} + a_m \delta \frac{\partial^2 T}{\partial x^2} , \tag{2}$$

$$\left. \frac{\partial T}{\partial x} \right|_{x=0} = 0 , \quad \left. \frac{\partial U}{\partial x} \right|_{x=0} = 0 , \tag{3}$$

$$- k \left. \frac{\partial T}{\partial x} \right|_{x=R} + \alpha \left(T_{\text{amb}} - T|_{x=R} \right) - (1 - \varepsilon) \, r \beta \rho \left(U|_{x=R} - U_{\text{eq}} \right) = 0 , \tag{4}$$

$$a_m \left. \frac{\partial U}{\partial x} \right|_{x=R} + a_m \delta \left. \frac{\partial T}{\partial x} \right|_{x=R} + \beta \left(U|_{x=R} - U_{\text{eq}} \right) = 0 , \tag{5}$$

$$T|_{t=0} = T_0 , \quad U|_{t=0} = U_0 . \tag{6}$$

The parameters a_m, ε, δ, and β characterize the process of moisture transfer. The moisture-diffusion coefficient a_m is an analog of the thermal-diffusivity coefficient a_q and characteritizes the velocity of moisture movement in the body. The mass-transfer coefficient β is an analog of the heat-transfer coefficient α. It characterizes the intensity of moisture transfer to the ambient medium. The phase-transition number ε is equal to the ratio of the amount of moisture moving in the body in the form of vapor to the amount of moisture moving in the form of liquid. The thermogradient coefficient δ is the ratio of the moisture-content difference in the body to the temperature difference in the absence of moisture transfer.

Expressions (1)-(6) are similar to the equations of [2] for the problem of heat and moisture transfer in a plate with the same initial conditions and differ from them by the presence of the third term on the right-hand side of Eq. (1), which takes into account the internal heat sources. Let us assume that they are uniformly distributed throughout the volume of the plate and that their intensity does not change with time.

In solving Eqs. (1) and (2), we use the Laplace transformation, just as was done in [2]. The procedure for solving this problem is similar to the one given in [2]. The differences are only, in the appearance of terms connected with the presence of the internal heat sources. Therefore we omit all intermediate mathematical computations and write the final result:

$$T(x, t) = T_{\text{amb}} + \frac{QR^2}{2k} \left(1 + \frac{2}{\text{Bi}_q} - \frac{x^2}{R^2} \right) - $$

$$- 2 \left(T_{\text{amb}} - T_0 \right) \sum_{n=1}^{\infty} \left(C_{2n} \cos \frac{\nu_1 \mu_n x}{R} - C_{1n} \cos \frac{\nu_2 \mu_n x}{R} \right) \exp \left(- t / \tau_n \right) , \tag{7}$$

$$U(x, t) = U_{\text{eq}} - \frac{QR^2 \delta}{2k} \left(1 - \frac{x^2}{R^2} \right) - 2 \left(U_0 - U_{\text{eq}} \right) \times$$

$$\times \sum_{n=1}^{\infty} \left[D_{2n} (1 - \nu_1^2) \cos \frac{\nu_1 \mu_n x}{R} - D_{1n} (1 - \nu_2^2) \cos \frac{\nu_2 \mu_n x}{R} \right] \exp (- t/\tau_n) , \quad (8)$$

where

$$C_{jn} = \frac{\varepsilon \,\mathrm{Ko}\, Q_{jn} + \left(1 - \varepsilon \,\mathrm{Ko}\, K_1 + \frac{\xi R^2}{a_q \mu_n}\right) P_{jn}}{\mu_n \psi_n} ;$$

$$D_{jn} = \frac{C_{jn}}{\varepsilon \,\mathrm{Ko}} , \quad j = 1, 2 ;$$

$$\psi_n = \nu_1 A_{1n} P_{2n} + \nu_2 B_{2n} Q_{1n} - \nu_2 A_{2n} P_{1n} - \nu_1 B_{1n} Q_{2n} ;$$

$$P_{jn} = (1 - \nu_j^2) \cos \nu_j \mu_n - \frac{\nu_j \mu_n (1 - \nu_j^2 + \mathrm{Fe})}{\mathrm{Bi}_m} \sin \nu_j \mu_n ;$$

$$Q_{jn} = [1 + (1 - \nu_j^2) K_1] \cos \nu_j \mu_n - \frac{\nu_j \mu_n}{\mathrm{Bi}_q} \sin \nu_j \mu_n ;$$

$$A_{jn} = \left[1 + (1 - \nu_j^2) K_1 + \frac{1}{\mathrm{Bi}_q} \right] \sin \nu_j \mu_n + \frac{\nu_j \mu_n}{\mathrm{Bi}_q} \cos \nu_j \mu_n ;$$

$$B_{jn} = \left(1 - \nu_j^2 + \frac{1 - \nu_j^2 + \mathrm{Fe}}{\mathrm{Bi}_m} \right) \sin \nu_j \mu_n + \frac{\nu_j \mu_n (1 - \nu_j^2 + \mathrm{Fe})}{\mathrm{Bi}_m} \cos \nu_j \mu_n ;$$

$$\nu_j^2 = \frac{1}{2} \left[\left(1 + \mathrm{Fe} + \frac{1}{\mathrm{Lu}} \right) + (-1)^j \sqrt{\left(\left(1 + \mathrm{Fe} + \frac{1}{\mathrm{Lu}} \right)^2 - \frac{4}{\mathrm{Lu}} \right)} \right] ;$$

$$K_1 = \frac{1 - \varepsilon}{\varepsilon} \mathrm{Lu} \frac{\mathrm{Bi}_m}{\mathrm{Bi}_q} ; \quad \xi = \frac{Q}{c\rho (T_{\mathrm{amb}} - T_0)} ; \quad \tau_n = \frac{R^2}{\mu_n^2 a_q} .$$

The coefficients μ_n are the roots of the equation

$$Q_{2n} P_{1n} - Q_{1n} P_{2n} = 0 .$$

For $Q = 0$ (superhigh-frequency heating is absent), the solution obtained coincides with the solution of [2].

Figures 1 and 2 show some results of calculations by formulas (7) and (8) with the following initial data: $R = 0.1$ m, $\rho = 600$ kg/m^3, $c = 1500$ J/(kg·deg), $k = 0.2$ W/(m·deg), $a_m = 3.33 \cdot 10^{-8}$ m^2/sec, $\delta = 3.75 \cdot 10^{-4}$ 1/deg, $r = 2.4 \cdot 10^6$ J/kg, $\varepsilon = 0.1$, $\alpha = 20$ W/(m^2·deg), $\beta = 3.33 \cdot 10^{-5}$ m/sec, $T_{\mathrm{amb}} = 50^{\mathrm{o}}$C, $T_0 = 20^{\mathrm{o}}$C, $U_0 = 0.24$, and $U_{\mathrm{eq}} = 0.1$. They approximately correspond to conditions for drying wood bars.

Figure 1a shows the distribution of the temperature and the moisture content in the plate at different instants of time in the absence of internal heat sources. The plate receives energy from the ambient medium, and its temperature gradually increases, always being higher at the edges than at the center. The temperature increase enhances the diffusion of moisture and its evaporation from the surface of the plate so that at the

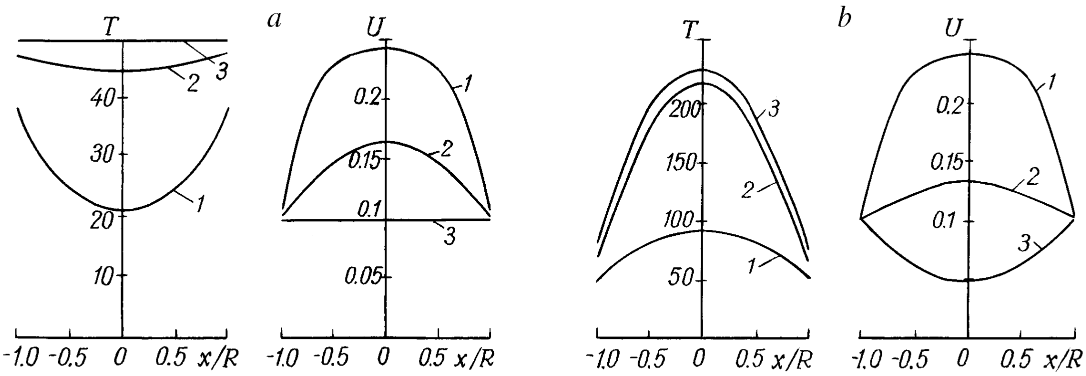

Fig. 1. Temperature and moisture-content distribution in the absence (a, Q = 0) and in the presence (b, Q = 6000 W/m^3) of internal heat sources: 1) t/τ_1 = 0.1; 2) 1; 3) 4. T, ^{o}C; U, kg/kg.

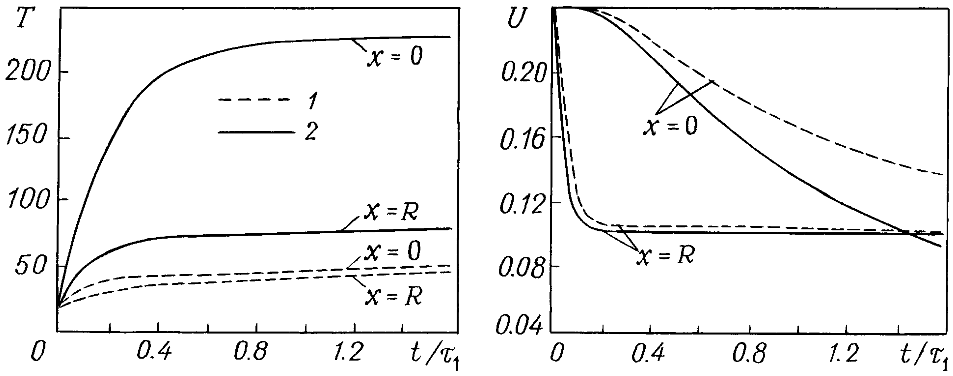

Fig. 2. Time dependence of the temperature and the moisture content: 1) Q = 0; 2) 6000 W/m^3.

plate's edges the moisture content is lower than at the center. The average moisture content gradually decreases, tending to the equilibrium value.

The time constant τ_1, which plays the most significant role among all τ_n, was selected as a time scale. Since the thickness of the plate is large, the thermal processes occur slowly – $\tau_1 = 1.26 \cdot 10^5$ sec ≈ 35 h.

The presence of internal heat sources substantially changes the character of the process of drying. However for this it is necessary that the amount of heat coming from them be comparable to the amount of heat coming from the ambient medium. Mathematically this means that the inequality

$$\frac{QR}{\alpha\,(T_{\text{amb}} - T_0)} \geq 1$$

must be fulfilled. With the above initial data this holds true if the density of the internal heat sources is $Q \geq 6000$ W/m^3.

Figure 1b shows the distribution of the temperature and the moisture content in the plate under these conditions. It can be seen from the figure that the temperature maximum is at the center now, and the average temperature is higher than in Fig. 1a. Both these factors contribute to intensification of the drying process (see Fig. 1b).

Figure 2 shows the time dependence of the temperature and the moisture content. One can also see here the role of internal heat sources in the intensification of the drying process. For Q = 6000 W/m^3, the moisture content at the center of the plate decreases from a value of 0.24 to 0.1 in the time $1.5\tau_1$, while for Q = 0 the value U = 0.1 is attained in the time $4\tau_1$.

Of interest are the expressions for the volume-average temperature and moisture content. Having averaged (7) and (8), we obtain

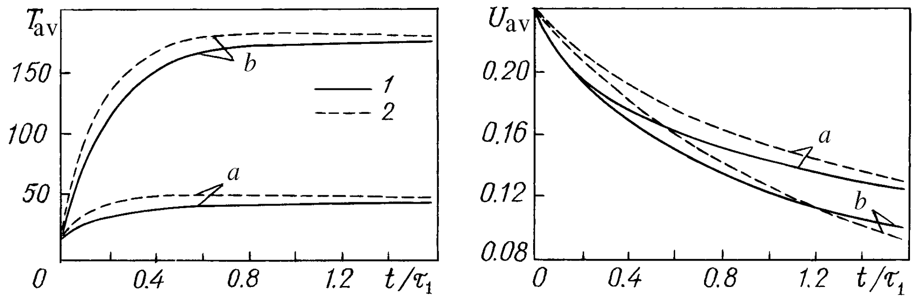

Fig. 3. Time dependence of the average values of the temperature and the moisture content: 1) according to (9) and (10); 2) according to (11) and (16); a) $Q = 0$; b) 6000 W/m^3.

$$T_{av}(t) = T_{amb} + \frac{QR}{\alpha}\left(1 + \frac{\mathrm{Bi}_q}{3}\right) - 2\,(T_{amb} - T_0) \sum_{n=1}^{\infty} M_n \exp(-t/\tau_n)\,, \tag{9}$$

$$U_{av}(t) = U_{eq} - \frac{QR^2\delta}{3k} - 2\,(U_0 - U_{eq}) \sum_{n=1}^{\infty} N_n \exp(-t/\tau_n)\,, \tag{10}$$

where

$$M_n = C_{2n} \frac{\sin \nu_1\mu_n}{\nu_1\mu_n} - C_{1n} \frac{\sin \nu_2\mu_n}{\nu_2\mu_n}\,,$$

$$N_n = D_{2n}(1 - \nu_1^2) \frac{\sin \nu_1\mu_n}{\nu_1\mu_n} - D_{1n}(1 - \nu_2^2) \frac{\sin \nu_2\mu_n}{\nu_2\mu_n}\,.$$

Graphs constructed from (9) and (10) are shown in Fig. 3 as solid lines. They confirm that the presence of internal heat sources substantially intensifies the process of drying. It can also be seen that the heat transfer occurs faster than the mass transfer. This is due to the fact that in this case the thermal-diffusivity coefficient of the substance is higher than the moisture-diffusion coefficient.

Expressions (7)-(10) have a complex form, and calculations by them are very time-consuming. For practical calculations, simplified formulas that can be used for rapid evaluation of the parameters of the drying process (first of all, its duration and the maximum temperature of heating the material) are of interest. To obtain them, let us analyze Eqs. (1) and (2).

The course of heating capillary-porous solid bodies with a low moisture content that are similar to those enumerated at the beginning of the paper depends little on their moisture. This becomes clear if Eq. (1) and the heat-conduction equation that lacks the term $\frac{\varepsilon r}{c}\frac{\partial U}{\partial t}$ are solved. Here the thermal time constants are determined by the expression $\tau_n = R^2/\mu_n^2 a_q$, where μ_n are the roots of the equation $\tan \mu_n = \mathrm{Bi}_q/\mu_n$ [7]. The main role is played by the time constant τ_1. The value of the root μ_1 can be determined from the following approximate formula:

$$\mu_1 = \sqrt{\left(\frac{\mathrm{Bi}_q}{1 + \frac{\mathrm{Bi}_q}{3}}\right)}\,.$$

The variation in the volume-average temperature is described well by the dependence

$$T(t) = T_0 + \left(T_{\text{amb}} - T_0 + \frac{Q\tau_q}{c\rho}\right)[1 - \exp(-t/\tau_q)]\,, \tag{11}$$

where

$$\tau_q = \tau_1 = \frac{c\rho R}{\alpha}\left(1 + \frac{\text{Bi}_q}{3}\right). \tag{12}$$

Figure 3 shows as dashed lines the time dependence of the temperature calculated by formula (11). It is seen that the approximate curves differ from the exact ones insignificantly.

Expression (11) is a solution of the heat-balance equation

$$\frac{dT}{dt} + \frac{T - T_{\text{amb}}}{\tau_q} = \frac{Q}{c\rho}\,. \tag{13}$$

Equation (2) describing the change in the moisture content with time represents a diffusion equation in which the role of the sources is played by the second term on the right-hand side. The time constants of this process are determined by a formula similar to the formula for the thermal time constants τ_q:

$$\tau_n = \frac{R^2}{\mu_n^2 a_m}\,,$$

in which μ_n are the roots of the equation $\tan \mu_n = \text{Bi}_m/\mu_n$. Since usually $\text{Bi}_m >> 1$,

$$\mu_n = \left(n - \frac{1}{2}\right)\frac{\pi}{2}\,, \quad n = 1, 2, 3, \ldots\,.$$

The time constant playing the most important role is

$$\tau_m = \tau_1 = \frac{4R^2}{\pi^2 a_m}\,.$$

The moisture-transfer process can be described by an equation similar to the heat-balance equation (13):

$$\frac{dU}{dt} + \frac{U - U_{\text{eq}}}{\tau_m} = a_m \delta \frac{\partial^2 T}{\partial x^2}\,. \tag{14}$$

From Eq. (1) without the second term on the right-hand side it follows that

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{a_q}\frac{\partial T}{\partial t} - \frac{Q}{k}\,.$$

Having substituted this relationship into expression (14) and having used Eq. (11) to calculate the derivative $\partial T/\partial t$, we obtain the following equation for the moisture content:

$$\frac{dU}{dt} + \frac{U}{\tau_m} = V + W \exp\left(-\frac{t}{\tau_q}\right), \tag{15}$$

where

$$V = \frac{U_{eq}}{\tau_m} - \frac{a_m \delta Q}{k}; \quad W = \frac{a_m \delta}{a_q \tau_q} \left(T_{amb} - T_0 + \frac{Q \tau_q}{c\rho} \right).$$

The solution of Eq. (15) with the initial condition $U(0) = U_0$ will be

$$U(t) = A - B \exp(-t/\tau_q) + (U_0 - A + B) \exp(-t/\tau_m), \qquad (16)$$

where

$$A = U_{eq} - \frac{4QR^2\delta}{\pi^2 k}; \quad B = \frac{a_m \delta \left(T_{amb} - T_0 + \frac{Q\tau_q}{c\rho} \right)}{a_q \left(1 - \frac{\tau_q}{\tau_m} \right)}.$$

The curves calculated by formula (16) are shown in Fig. 3 as dashed lines. One can see that they differ little from the results of calculation by the more rigorous formula (10).

Thus, for a rough evaluation of the heating temperature and the drying time, use can be made of formulas (11) and (16). It is only necessary to be aware of the limits of their application. The formulas hold true in cases where the moisture content of the material is comparatively low and therefore does not affect substantially the process of heating the body. Such materials are ceramics, clay, gypsum, wood, grain products, etc.

Relations (7)-(10) are mathematically more rigorous. However the thermophysical parameters of capillary-porous bodies are known, as a rule, only roughly, with a large error. Therefore formulas (7)-(10) do not give a much more accurate evaluation of the actual process than approximate relations (11) and (16).

NOTATION

a_m, moisture-diffusion coefficient, m^2/sec; a_q, thermal-diffusivity coefficient, m^2/sec; c, specific heat, J/(kg·deg); k, thermal-conductivity coefficient, W/(m·deg); Q, volume density of the heat sources, W/m^3; R, half of the plate thickness, m; r, specific heat of evaporation, J/kg; T, running temperature of the body, ^oC; T_{amb}, temperature of the ambient medium, ^oC; T_0, initial temperature of the body, ^oC; t, time, sec; U, running moisture content in the body, kg/kg; U_{eq}, equilibrium moisture content, kg/kg; U_0, initial moisture content in the body, kg/kg; x, running coordinate, m; α, heat-transfer coefficient, W/(m^2·deg); β, mass-transfer coefficient, m/sec; δ, thermogradient coefficient, 1/deg; ε, dimensionless phase-transition number; τ_q and τ_m, time constants of the process of change of the temperature and the moisture content, sec; $Bi_q = \alpha R/k$, dimensionless Biot heat-transfer number; $Bi_m = \beta R/a_m$, dimensionless Biot mass-transfer number; $Fe = \varepsilon r\delta/c$, dimensionless Fedorov number; $Ko = (r(U_0 - U_{eq}))/(c(T_{amb} - T_0))$, dimensionless Kossovich number; $Lu = a_m/a_q$, dimensionless Luikov number.

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